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AN ADAPTIVE EXPECTATIONS MODEL OF THE EAST-WEST ARMS RACE

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I. INTRODUCTION

Interactive specifications of military arms race models have been prominent for some time. Boulding [1], Intrilligator [2], and McGuire [4] among others[1] have made recent graphical and mathematical contributions; all extend the economic theory of duopoly to the arms race process. While all are indebted to Richardson [6], few have attempted to specify and then empirically test a formal arms race model. In this study I shall borrow from econometric models of market processes[2] and estimate and simulate the model for the NATO and Warsaw Pact alliances.

II. THE MODEL

The basic hypothesis entertained is that, on the basis of past history, a nation, N, forms an anticipation or expectation of his adversary's defense expenditures, W, and would like to adjust its expenditures in accordance with the relation:

$$N_t^* = a + bW_t' \tag{1}$$

where the asterisk denotes desired levels of expenditure and the prime denotes expected or anticipated levels of expenditure. The parameter a reflects minimal amounts of defense expenditures which nation N requires and the parameter b reflects the 'expenditure-reaction' of N to W.

We may describe the movement of N towards its desired level by the relation:

$$N_t - N_{t-1} = \delta(N_t^* - N_{t-1}), \qquad 0 < \delta < 1 \tag{2}$$

Thus the observed change in N's expenditure is an adjustment towards desired level N_t^*.

Furthermore, let us specify that N's expectation of W's expenditures follow this adjustment process:

$$W_t' - W_{t-1}' = \beta(W_t - W_{t-1}') \tag{3}$$

Thus N is reacting to expected expenditures of W and is attempting to adjust its

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[1] For a more complete bibliography and a discussion of arms race research strategies, see Singer [7].

[2] The model developed is an extension of Nerlove's [5] model of agricultural markets.

expenditures to a desired level.

To obtain sample estimates of a and b, we must recast the model in terms of observables. Fortunately, through a series of transformations, we can respecify N_t in terms of W_t, N_{t-1}, and N_{t-2}.

From (2) we have:

$$N_t = \delta(N_t^* - N_{t-1}) + N_{t-1} \tag{4}$$

Substituting (1) into (4) yields);

$$N_t = N_{t-1} + \delta(a + bW_t') - \delta N_{t-1}$$

or:

$$N_t = (1 - \delta)N_{t-1} + a\delta + bW_t' \tag{5}$$

Substituting (3) into (5):

$$N_t = (1 - \delta)N_{t-1} + a\delta + b\delta[\beta(W_t - W_{t-1}') + W_{t-1}']$$

or:

$$N_t = (1 - \delta)N_{t-1} + a\delta + b\delta\beta W_t + b\delta(1 - \beta)W_{t-1}' \tag{6}$$

Solving (5) for time period $t - 1$ for W_{t-1}' and substituting this into (6) yields:

$$N_t = (1 - \delta)N_{t-1} + a\delta + b\delta\beta W_t + b\delta(1 - \beta)\left\{\frac{N_{t-1} - a\delta - (1 - \delta)N_{t-2}}{b\delta}\right\} \tag{7}$$

which reduces to:

$$N_t = a\delta\beta + b\beta\delta W_t + (1 - \beta - \delta)N_{t-1} - (1 - \beta)(1 - \delta)N_{t-2} \tag{8}$$

Now if W reasons as N has, then the symmetrical relation:

$$W_t^* = c + dN_t' \tag{9}$$

will lead via an analogous deduction to a statement similar to (8) but now in parameters c, d, α and γ:

$$W_t = c\alpha\gamma + d\alpha\gamma N_t + (2 - \alpha - \gamma)W_{t-1} - (1 - \alpha)(1 - \gamma)W_{t-2} \tag{10}$$

In terms of observables, N and W then depend contemporaneously on each other and on lagged values of themselves.

III. ESTIMATION PROCEDURE AND DATA

To obtain sample estimates of the model's parameters, we apply least squares regression analysis. In particular, we estimate:

$$N_t = \theta_1 + \theta_2 W_t + \theta_3 N_{t-1} + \theta_4 N_{t-2} + e_t \tag{11}$$

$$W_t = \pi_1 + \pi_2 N_t + \pi_3 W_{t-1} + \pi_4 W_{t-2} + v_t \tag{12}$$

the θ's and π's are regression coefficients; e_t and v_t are random disturbances terms with zero means and constant variances. To insure that two important assumptions of regression analysis are met: $\varepsilon(e_t W_t) = \varepsilon(v_t N_t) = 0$ and $\varepsilon(e_t v_t) = 0$, we perform three stage least squares regression on (11) and (12).[3]

We may derive parameter estimates from the calculated regression coefficients as follows:[4]

(13a) $\hat{\theta}_1 = (a\beta\delta)$ (13c) $\hat{\theta}_3 = (2 - \beta - \delta)$

(13b) $\hat{\theta}_2 = (b\beta\delta)$ (13d) $\hat{\theta}_4 = [(1 - \beta)(1 - \delta)]$

Adding $\hat{\theta}_3$ to $\hat{\theta}_4$, we have:

$$\hat{\theta}_3 + \hat{\theta}_4 = 1 - \beta\delta \qquad \text{or:}$$

$$\beta\delta = 1 - \hat{\theta}_3 - \hat{\theta}_4 \tag{14}$$

Substituting (14) into (13a) and (13b), we find for a and b respectively:

$$\hat{a} = \frac{\hat{\theta}_1}{1 - \hat{\theta}_3 - \hat{\theta}_4} \tag{15a}$$

$$\hat{b} = \frac{\hat{\theta}_2}{1 - \hat{\theta}_3 - \hat{\theta}_4} \tag{15b}$$

Analogously:

$$\hat{c} = \frac{\hat{\pi}_1}{1 - \hat{\pi}_3 - \hat{\pi}_4} \tag{16a}$$

$$\hat{d} = \frac{\hat{\pi}_2}{1 - \hat{\pi}_3 - \hat{\pi}_4} \tag{16b}$$

Since β and δ enter the model symmetrically, we can not solve (13a) through (13d) to obtain unique values of each. However, we can derive a quadratic statement for each by adding and rearranging (13c) and (13d):

$$\beta = \frac{(2 - \hat{\pi}_3) \pm \sqrt{\hat{\pi}_3^2 + 4\hat{\pi}_4}}{2} \tag{17a}$$

$$\delta = \frac{(2 - \hat{\pi}_3) \mp \sqrt{\hat{\pi}_3^2 + 4\hat{\pi}_4}}{2} \tag{17b}$$

Similar results of course obtain for α and γ.

Data for the Nato and Warsaw Pact alliances are from the Stockholm International Peace Research Institute's *Yearbook of World Armaments and Disarmament 1968/9* [8]. Figures are in billions of 1960 U.S. dollars and utilize Benoit-Lubell exchange rates for the Warsaw Pact. Table 1 presents the series, 1949–1969.[5]

[3] See Johnston [3], pp. 266–268.

[4] The carrot (' ^ ') denotes calculated values.

[5] For a complete discussion of the data creation, see [8], pp. 194–199.

TABLE 1: Nato and Warsaw Pact Defense Expenditures in Billions of 1960 U.S. Dollars and Using Benoit-Lubell Exchange Rates

Year	Nato (N)	Warsaw Pact (W)
1949	23.905	21.357
1950	26.692	22.231
1951	50.231	25.448
1952	68.487	28.452
1953	70.287	28.166
1954	61.711	26.381
1955	58.985	27.976
1956	60.682	25.917
1957	62.382	25.856
1958	60.811	25.204
1959	62.427	25.508
1960	61.335	25.522
1961	63.689	31.371
1962	69.101	34.424
1963	68.935	37.540
1964	67.573	36.106
1965	67.280	34.892
1966	76.776	36.638
1967	86.608	39.532
1968	87.755	45.803
1969	87.443*	48.938*

* Tentative

III. ESTIMATION AND SIMULATION RESULTS

Three stage least squares estimates of (11) and (12) were computed to be[6,7]:

$$\text{(18)} \quad \hat{N}_t = \underset{(4.719)}{21.9600} + \underset{(3.753)}{.8046W_t} + \underset{(3.723)}{.6612N_{t-1}} - \underset{(-3.065)}{.3714N_{t-2}} \qquad \begin{matrix} R^2 = .9155 \\ \sigma = 3.2404 \end{matrix}$$

$$\text{(19)} \quad W_t = \underset{(-.3949)}{-5.0820} + \underset{(.5492)}{.1879N_t} + \underset{(1.1480)}{.7739W_{t-1}} + \underset{(.0392)}{.0239W_{t-2}} \qquad \begin{matrix} R^2 = .8835 \\ \sigma = 2.8098 \end{matrix}$$

The terms in parentheses are t ratios (the ratio of regression coefficient to its standard error) which allow us to test the null hypothesis that the particular coefficient is not statistically significant from zero. All four coefficients in (18) are significantly different from zero at the 95% level, and all four in (19) are not significantly different from zero at the 95% level. The adaptive-expectations hypothesis seems then to fit observed Nato behavior quite well. While the t-ratios for (19) are generally poor, the R^2 and standard error of forecast (σ) are quite strong; this suggests that

[6] Computations were performed at the Triangle Universities Computation Center on the IBM O/S Model 360–75 using a double precision version of Zellner-Stroud's "Two-Three Stage Least Squares," Madison, Wisconsin, February, 1967.

[7] Reported R^2 and σ's refer to second stage results.

N_t, W_{t-1}, and W_{t-2} are collinear.[8] Utilizing the calculated regression coefficients in (18) and (19), we find the parameters of the model to be (in billions of 1960 U.S. dollars):

$$\text{(20)} \quad \hat{a} = 30.9209 \qquad \hat{c} = -25.1323$$

$$\hat{b} = 1.1329 \qquad \hat{d} = .9292$$

Our model of the arms race then becomes:

$$\text{(21)} \quad N_t^* = 30.9209 + 1.13292W_t'$$

$$\text{(22)} \quad W_t^* = -25.1323 + .92923N_t'$$

Apparently Nato is willing to spend \$1.13 to the dollar of expected Warsaw Pact expenditures while the Warsaw Pact is willing to spend only .93 to the expected Nato dollar. The negative constant or minimum amount of Warsaw expenditures is puzzling, though $\hat{c}$ is based on a set of statistically insignificant regression coefficients.

Of particular interest are forecasts of Nato and Warsaw Pact expenditures in the next three decades. To generate such forecasts, we solve (18) and (19) so that N_t and W_t are functions of lagged N and W's and use predicted N_t and W_t's as inputs into the model for N_{t+1} and W_{t+1}. To make the simulation more realistic, we disturb the model randomly each time period by drawing a random digit between -1 and $+1$ and adding the product of this times σ to the forecast for each equation. Table 2 presents the results of this stochastic simulation for the period of 1970–2000; forecasts are in billions of constant 1960 U.S. dollars. The simulation suggests an upward trend in the east-west arms race with occasional dips. Interestingly, it suggests a relative decline in Warsaw expenditures in 1971 but no relative decline in Nato expenditures until 1974. Other dips occur but the general upward trend is apparent.

IV. CONCLUSIONS

An interactive arms race model has been developed which relates desired defense expenditure levels to anticipated levels of the adversary. When applied to Nato and Warsaw Pact data, the estimated model suggests Nato expenditure reactions of \$1.13 to the Warsaw dollar and Warsaw expenditure reactions of \$.93 to the Nato dollar. Stochastic simulation of the model for the next three decades yields a slow upward rise in Nato and Warsaw expenditure levels with occasional one-year relative declines in each.

[8] See Johnston [3], pp. 201–207.

TABLE 2: Stochastic Simulation of Adaptive Expectations Model of Nato-Warsaw Pact Arms Race (Forecasts in Billions of 1960 U.S. Dollars)

Year	Predicted Nato Expenditure	Predicted Warsaw Pace Expenditure
1970	87.739	48.373
1971	88.548	47.948
1972	89.107	49.232
1973	89.050	50.118
1974	87.236	49.494
1975	84.297	50.242
1976	87.231	50.701
1977	87.774	53.212
1978	89.523	56.888
1979	96.590	56.404
1980	101.332	58.328
1981	104.024	61.470
1982	106.732	62.233
1983	103.931	62.291
1984	104.929	61.753
1985	101.807	65.700
1986	103.774	66.628
1987	108.774	67.408
1988	111.451	68.941
1989	111.243	72.096
1990	111.138	76.041
1991	119.121	79.704
1992	124.661	80.897
1993	128.077	81.460
1994	130.353	83.886
1995	129.862	87.546
1996	133.752	87.171
1997	132.530	88.411
1998	135.291	89.342
1999	136.087	89.311
2000	135.926	90.772

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